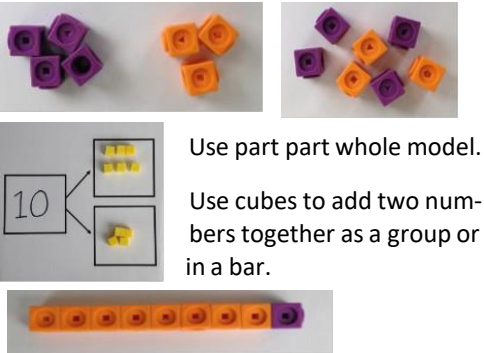
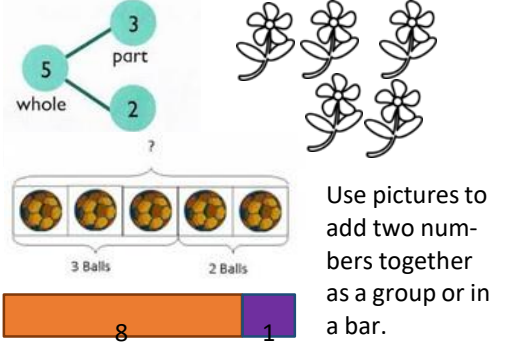
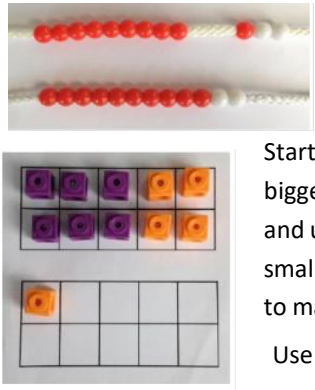
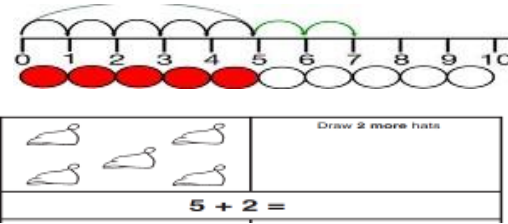


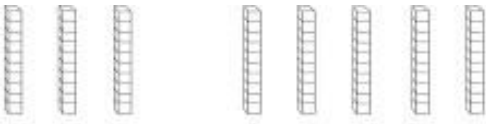
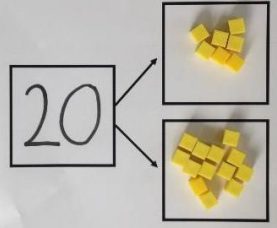
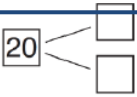
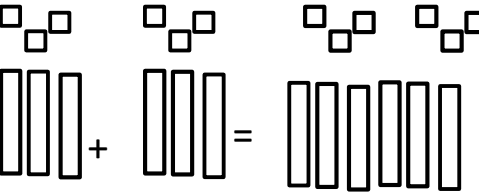
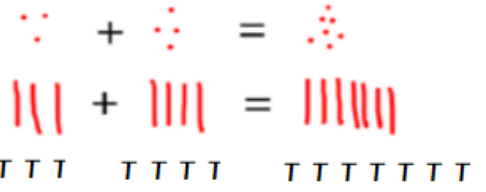
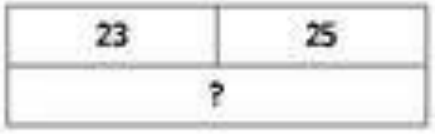


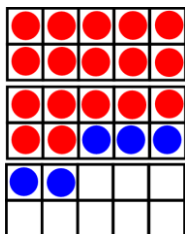
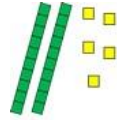
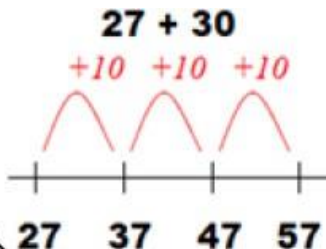
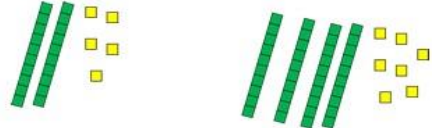
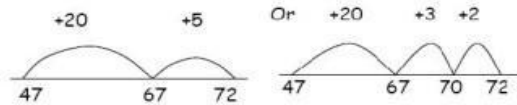
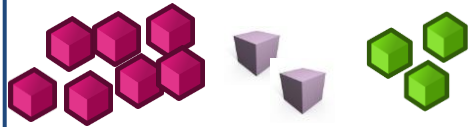
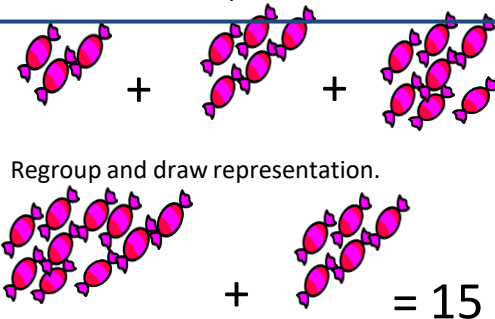
The development of methods is cumulative: each method builds upon a previously taught method.

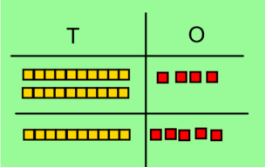
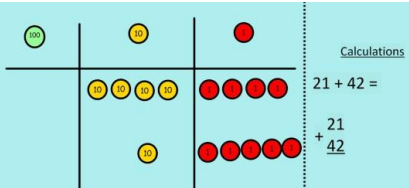
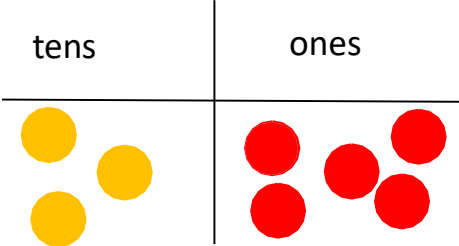
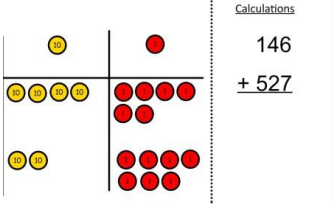
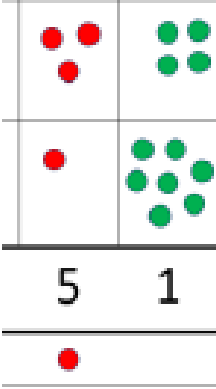
Strategies and methods must therefore be introduced in the order found in the calculation policy.

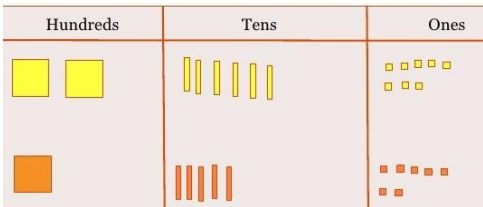
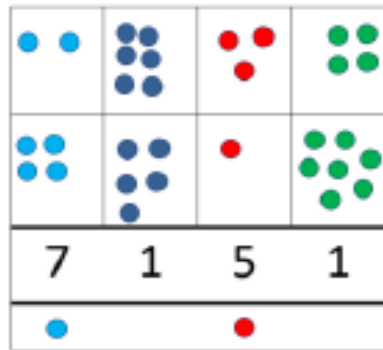
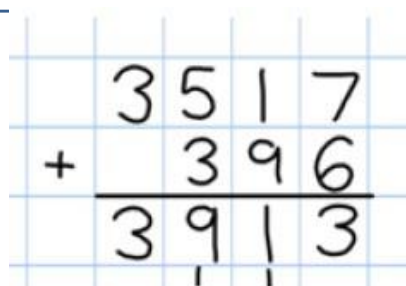
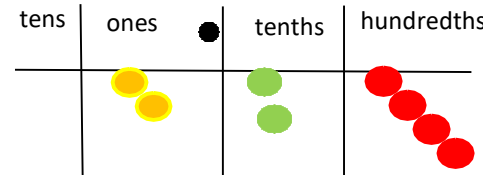
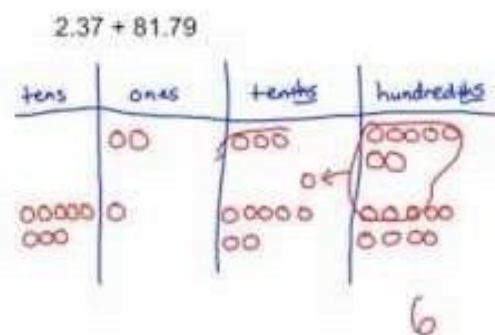
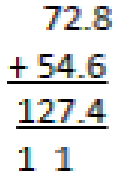
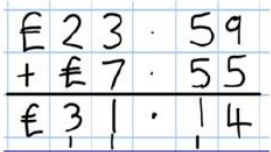
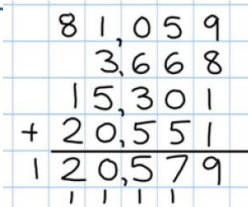
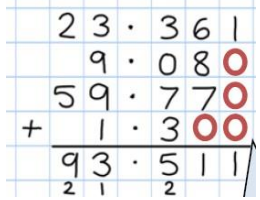
If your stream have mastered the methods assigned to their year group, introduce methods from the year above.

Objective & Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part- whole model	 Use part part whole model. Use cubes to add two numbers together as a group or in a bar.	 Use pictures to add two numbers together as a group or in a bar.	$4 + 3 = 7$  Use the part-part whole diagram as shown above to move into the abstract. $10 = 6 + 4$
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	$12 + 5 = 17$  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	$12 + 5 = 17$ IIII Draw lines underneath the 2nd number and count on similar as practised on the bead string.
Regrouping to make 10. <i>This is an essential skill for column addition later.</i>	 Start with the bigger number and use the smaller number to make 10. Use ten frames.	 Use pictures to make 10 and then count on.	$7 + 4 = 11$ $7 + \underline{\quad} = 11$ IIII If I am at seven, how many more do I need to make 10. How many more do I add on now? Children draw lines to count on one by one.
Represent & use number bonds and related subtraction facts within 20	 2 more than 5.	 5 + 2 =	Emphasis should be on the language '1 more than 5 is equal to 6.' '2 more than 5 is 7.' '8 is 3 more than 5.'

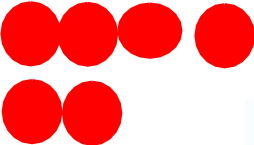
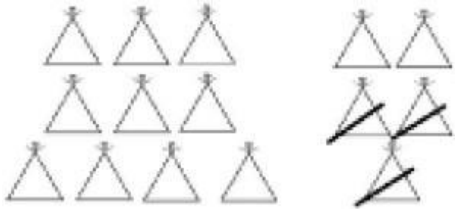
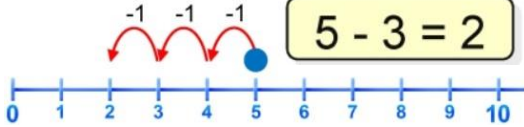
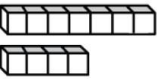
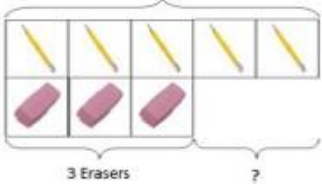
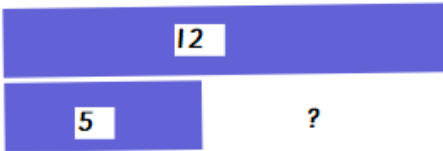
Objective & Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten	$50 = 30 + 20$  Model using dienes and bead strings	 $3 \text{ tens} + 5 \text{ tens} = \underline{\hspace{2cm}} \text{ tens}$ $30 + 50 = \underline{\hspace{2cm}}$ Use representations for base ten.	$20 + 30 = 50$ T T T Jot down T to represent the Tens and count on in tens. $70 = 50 + 20$ T T 50.....60, 70 $40 + \square = 60$ T T
Use known number facts Part part whole	 Children explore ways of making numbers within 20	 $\square + \square = 20$ $20 - \square = \square$ $\square + \square = 20$ $20 - \square = \square$	$\square + 1 = 16$ $16 - 1 = \square$ $1 + \square = 16$ $16 - \square = 1$
Using known facts	$\begin{array}{c} \square \square \\ \square \square \end{array} + \begin{array}{c} \square \square \\ \square \square \end{array} = \begin{array}{c} \square \square \square \square \\ \square \square \square \square \end{array}$ 	 $T T T$ $T T T T$ $T T T T T T T$ Children draw representations of H,T and O	$3 + 4 = 7$ <i>leads to</i> $30 + 40 = 70$ <i>leads to</i> $300 + 400 = 700$
Bar model	 $3 + 4 = 7$	 $7 + 3 = 10$	 $23 + 25 = 48$

Objective & Strategy	Concrete	Pictorial	Abstract				
Add a two digit number and ones	 $17 + 5 = 22$ Use ten frame to make 'magic ten' Children explore the pattern. $17 + 5 = 22$ $27 + 5 = 32$	$17 + 5 = 22$ Use part part whole and number line to model.	$17 + 5 = 22$ IIII Count on from 17...18, 19, 20, 21, 22 How many lines did you draw? 5 Explore related facts $17 + 5 = 22$ $5 + 17 = 22$ $22 - 17 = 5$ $22 - 5 = 17$ <table border="1"><tr><td colspan="2">22</td></tr><tr><td>17</td><td>5</td></tr></table>	22		17	5
22							
17	5						
Add a 2 digit number and tens	 $25 + 10 = 35$ Explore that the ones digit does not change	$27 + 30$ $+10 +10 +10$  $27 + 10 = 37$ T Jot letters in order to count on. $27 + 20 = 47$ T T $27 + \square = 57$ T T T					
Add two 2-digit numbers	 Model using dienes , place value counters and numicon	$+20 +5$ Or $+20 +3 +2$  Use number line and bridge ten using part whole if necessary.	$20 + 40 = 60$ $5 + 7 = 12$ $60 + 12 = 72$				
Add three 1-digit numbers	 Combine to make 10 first if possible, or bridge 10 then add third digit	 Regroup and draw representation. $= 15$	$4 + 7 + 6 = 10 + 7$ $= 17$ Combine the two numbers that make/ bridge ten then add on the third.				

Objective &	Concrete	Pictorial	Abstract
Strategy Column Addition—no regrouping (friendly numbers) Add two or three 2 or 3-digit numbers.	 Model using Dienes or numicon Add together the ones first, then the tens.  Move to using place value counters	Children move to drawing the counters using a tens and one frame. 	$\begin{array}{r} 223 \\ + 114 \\ \hline 337 \end{array}$ Add the ones first, then the tens, then the hundreds.
Column Addition with regrouping.	Exchange ten ones for a ten. Model using numicon, pv counters or dienes  Calculations $\begin{array}{r} 146 \\ + 527 \\ \hline \end{array}$	 Children can draw a representation of the grid to further support their understanding, carrying the ten <u>underneath</u> the line	$\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$

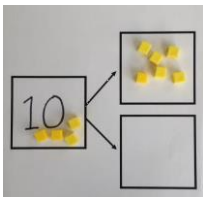
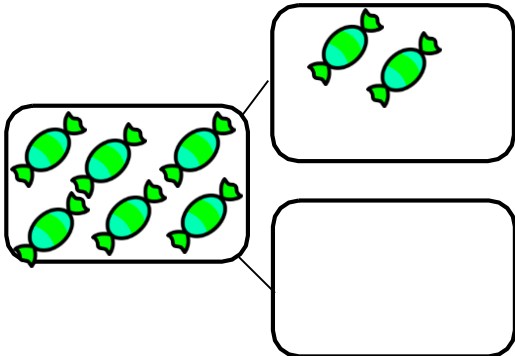
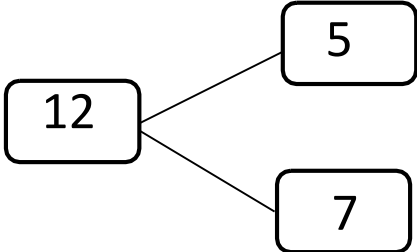
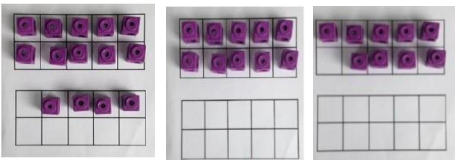
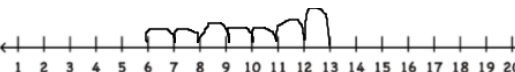
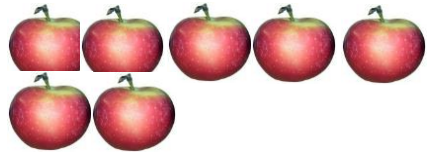
Objective & Strategy	Concrete	Pictorial	Abstract
Y4—add numbers with up to 4 digits	As per year 3, children use dienes if necessary. 	 Draw representations using pv grid.	 Continue from previous work to carry hundreds as well as tens. Relate to money and measures.
Y5—add numbers with more than 4 digits. Add decimals with 2 decimal places, including money.	As year 4  Introduce decimal place value counters and model exchange for addition.		 
Y6—add several numbers of increasing complexity Including adding money, measure and decimals with different numbers of decimal points.	As Y5	As Y5	 Insert zeros for place holders. 

Y4-6 ADDITION +

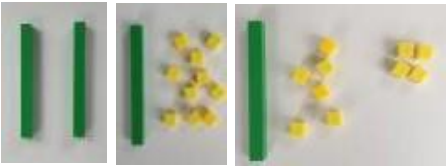
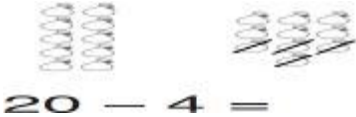
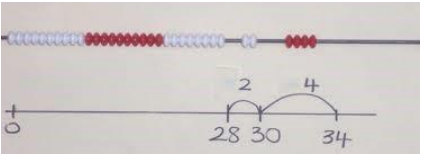
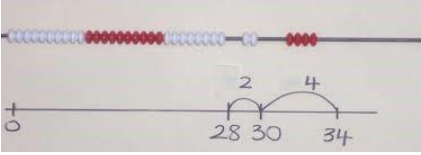
Objective & Strategy	Concrete	Pictorial	Abstract
Taking away ones.	Use physical objects, counters, cubes etc to show how objects can be taken away.  $6 - 4 = 2$  $4 - 2 = 2$	 $15 - 3 = 12$ Cross out drawn objects to show what has been taken away.	$7 - 4 = 3$  Jot the lines for the first number as it's subtraction.
Counting back	 Move objects away from the group, counting backwards.  Move the beads along the bead string as you count backwards.	 $5 - 3 = 2$ Count back in ones using a number line.	Put 13 in your head, count back 4. What number are you at? $13 - 4$ Jot the ones.  We cannot cross out 4 lines; therefore, we need to use the tens. $13 - 4$  Then, cross out 4. We're left with 7.
Find the Difference	Compare objects and amounts  'Seven is 3 more than four' 'I am 2 years older than my sister'  5 Pencils 3 Erasers ? Lay objects to represent bar model.	Count on using a number line to find the difference. 	Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister? $12 - 5$ 

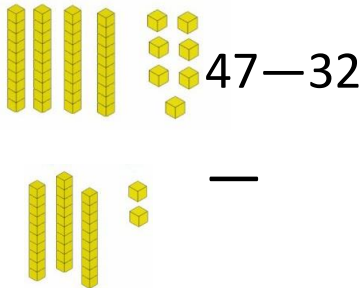
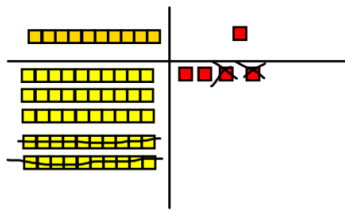
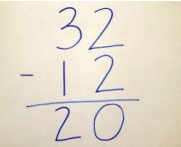
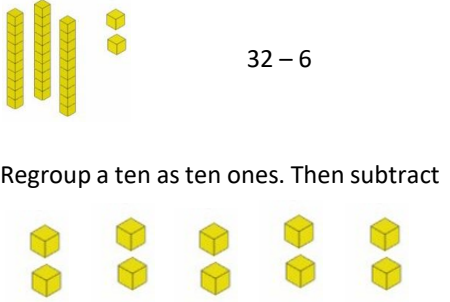
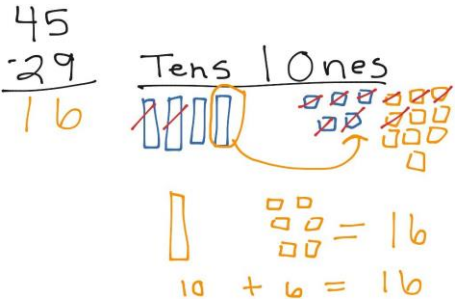
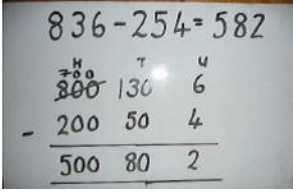
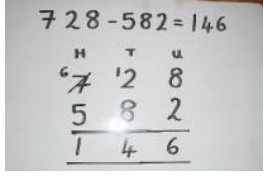
Y1 SUBTRACTION -

Y1 SUBTRACTION -

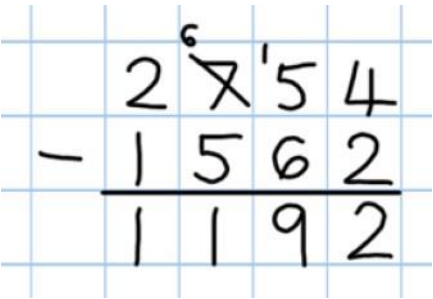
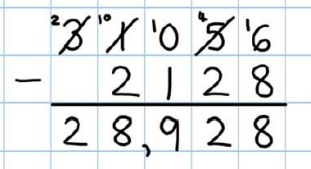
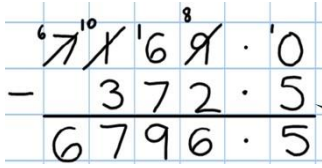
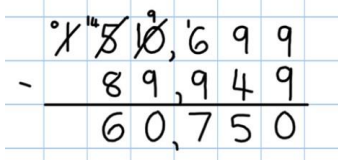
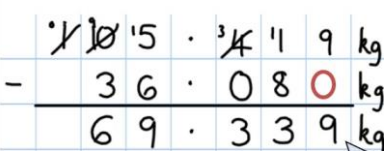
Objective & Strategy	Concrete	Pictorial	Abstract
Represent and use number bonds and related subtraction facts within 20 Part Part Whole model	 Link to addition. Use PPW model to model the inverse. If 10 is the whole and 6 is one of the parts, what's the other part? $10 - 6 = 4$	 Use pictorial representations to show the part.	Move to using numbers within the part whole model. 
Make 10	$14 - 9$  Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.	$13 - 7$ 	$16 - 8$ How many do we take off first to get to 10? How many left to take off?
Bar model	 $5 - 2 = 3$		 $10 = 8 + 2$ $10 = 2 + 8$ $10 - 2 = 8$ $10 - 8 = 2$

Y2 SUBTRACTION-

Objective & Strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	 Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'	 $20 - 4 =$	$20 - 4 = 16$
Partitioning to subtract without regrouping. <i>'Friendly numbers'</i>	$34 - 13 = 21$  Use Dienes to show how to partition the number when subtracting without regrouping.	Children draw representations of Dienes and cross off.  $43 - 21 = 22$	$43 - 21 = 22$ $\begin{array}{r} 43 \\ -21 \\ \hline 22 \end{array}$
Make ten strategy <i>Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.</i>	 $34 - 28$ Use a bead bar or bead strings to model counting to next ten and the rest.		$93 - 76 = 17$

Objective & Strategy	Concrete	Pictorial	Abstract
Column subtraction without regrouping (friendly numbers)	 $47 - 32$ Use base 10 or Numicon to model	 $\begin{array}{r} 54 \\ - 22 \\ \hline 32 \end{array}$ Calculations Draw representations to support understanding	 Intermediate step may be needed to lead to clear subtraction understanding.
Column subtraction with regrouping	 $32 - 6$ Regroup a ten as ten ones. Then subtract	 Children may draw base ten or PV counters and cross off.	 Begin by partitioning into pv columns Transition after 2 days.  Then move to formal method.

Y3 SUBTRACTION -

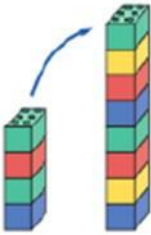
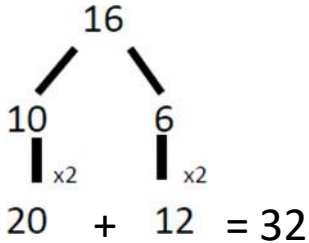
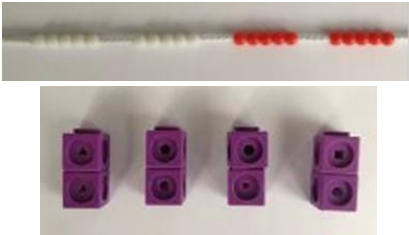
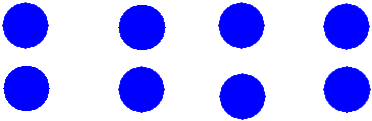
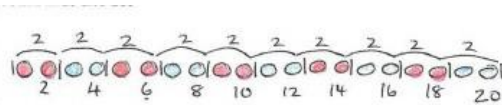
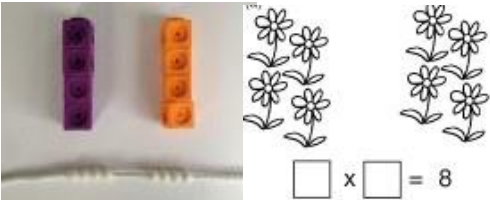
Objective & Strategy	Concrete	Pictorial	Abstract
Subtracting tens and ones Year 4 subtract with up to 4 digits. <i>Introduce decimal subtraction through context of money</i>	As per year 3, children to use concrete resources where necessary.	Children to draw pv counters and show their exchange—see Y3	 Use the phrase 'take and make' for exchange
Year 5- Subtract with at least 4 digits, including money and measures. <i>Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal</i>	As per year 3 and 4, children to use concrete resources where necessary.	Children to draw pv counters and show their exchange—see Y3	 Use zeros for place-holders. 
Year 6—Subtract with increasingly large and more complex numbers and decimal values.			 

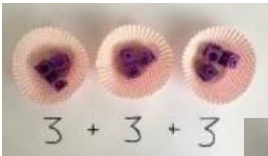
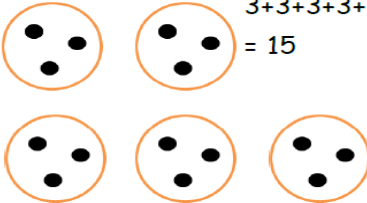
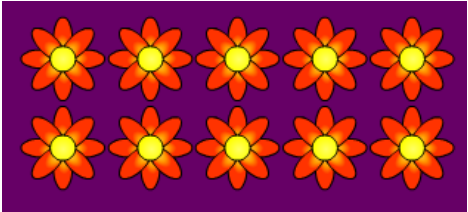
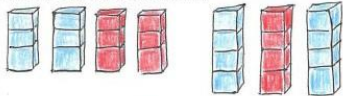
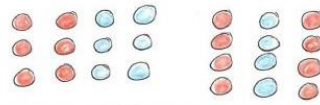
Y4-6

SUBTRACTION -

Y1

MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling  double 4 is 8 $4 \times 2 = 8$	Draw pictures to show how to double numbers Double 4 is 8 	Partition a number and then double each part before recombining it back together.  16 10 6 $\downarrow \times 2$ $\downarrow \times 2$ $20 + 12 = 32$
Counting in multiples	Count the groups as children are skip counting, children may use their fingers as they are skip counting. 	 Children make representations to show counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30
Making equal groups and counting the total	 Use manipulatives to create equal groups.	Draw  to show $2 \times 3 = 6$ Draw and make representations	$2 \times 4 = 8$

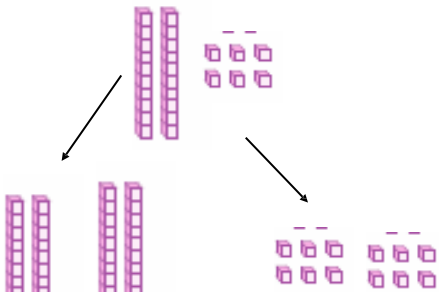
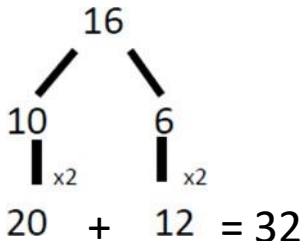
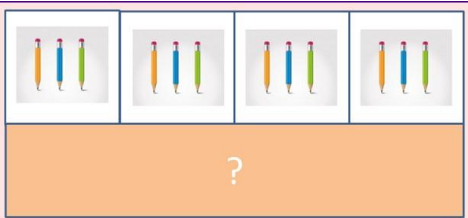
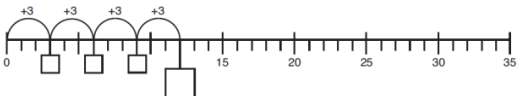
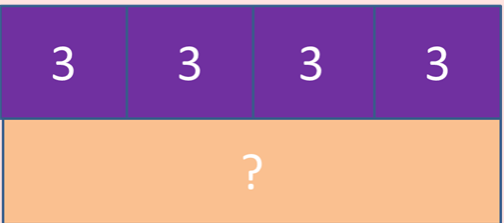
Objective & Strategy	Concrete	Pictorial	Abstract
Repeated addition	 	Use pictorial including number lines to solve problem There are 3 sweets in one bag. How many sweets are in 5 bags altogether? $3+3+3+3+3 = 15$  	Write addition sentences to describe objects and pictures.  $2 + 2 + 2 + 2 + 2 = 10$
Understanding arrays	Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc. 	Draw representations of arrays to show understanding.  	$3 \times 2 = 6$ $2 \times 5 = 10$

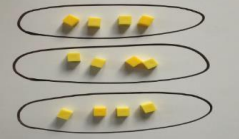
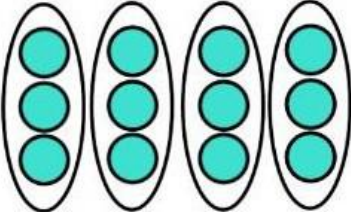
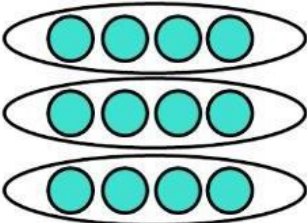
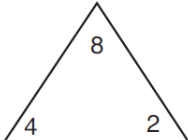
Y1

MULTIPLICATION X

Y2

MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Model doubling using dienes and PV counters.  $40 + 12 = 52$	Draw pictures and representations to show how to double numbers	Partition a number and then double each part before recombining it back together.  $20 + 12 = 32$
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.  $5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40$  	Number lines, counting sticks and bar models should be used to show representation of counting in multiples.    	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25, 30 $4 \times 3 = \square$

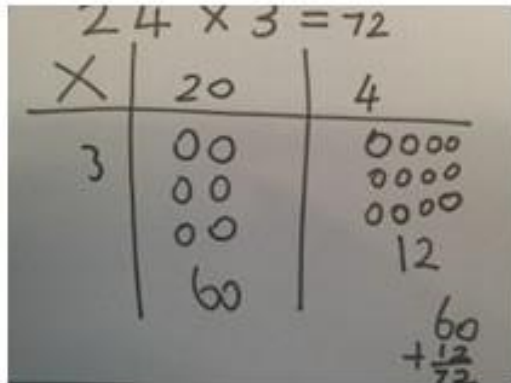
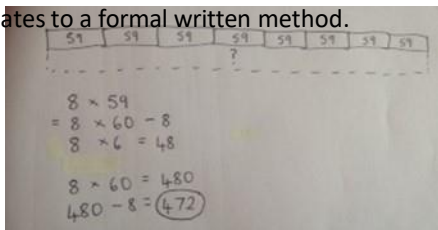
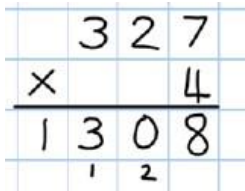
Objective & Strategy	Concrete	Pictorial	Abstract
Multiplication is commutative	Create arrays using counters and cubes and Numicon.   Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.  	Use representations of arrays to show different calculations and explore commutativity.  	$12 = 3 \times 4$ $12 = 4 \times 3$ Use an array to write multiplication sentences and reinforce repeated addition.  $5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$
Using the Inverse <i>This should be taught alongside division, so pupils learn how they work alongside each other.</i>		 □ × □ = □ □ × □ = □ □ ÷ □ = □ □ ÷ □ = □	$2 \times 4 = 8$ $4 \times 2 = 8$ $8 \div 2 = 4$ $8 \div 4 = 2$ $8 = 2 \times 4$ $8 = 4 \times 2$ $2 = 8 \div 4$ $4 = 8 \div 2$ Show all 8 related fact family sentences.

Y3

MULTIPLICATION X

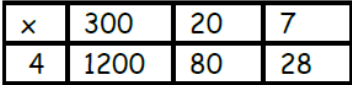
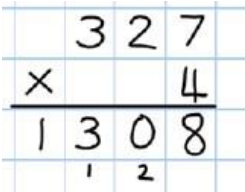
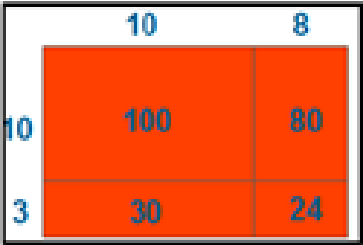
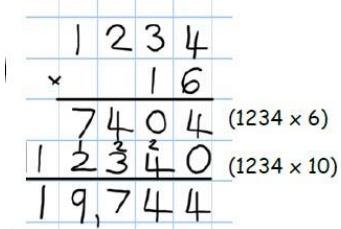
Objective & Strategy	Concrete	Pictorial	Abstract
Grid method	Show the links with arrays to first introduce the grid method. 10 3 4 4 rows of 10 4 rows of 3 Move onto base ten to move towards a more compact method. T U 4 rows of 13 Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows Calculations 4 x 126 Add up each column, starting with the ones Calculations 4 x 126 Then you have your answer.	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below. 24 x 3 = 72 3 20 4 60 12 60 72 Bar model are used to explore missing numbers 4 x = 20 20 4	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. x 30 5 7 210 35 210 + 35 = 245 Moving forward, multiply by a 2 digit number showing the different rows within the grid method. 10 8 10 3 100 80 30 24

Y4 MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract								
Grid method recap from year 3 for 2 digits x 1 digit Move to multiplying 3 digit numbers by 1 digit.	Children to use concrete methods taught in year 3 if necessary.	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below. 	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. <table border="1" data-bbox="1547 367 1881 466"><tr><td>x</td><td>30</td><td>5</td></tr><tr><td>7</td><td>210</td><td>35</td></tr></table> 210 + 35 = 245	x	30	5	7	210	35		
x	30	5									
7	210	35									
Column multiplication		<table border="1" data-bbox="1008 898 1359 984"><tr><td>x</td><td>300</td><td>20</td><td>7</td></tr><tr><td>4</td><td>1200</td><td>80</td><td>28</td></tr></table> The grid method may be used to show how this relates to a formal written method. 	x	300	20	7	4	1200	80	28	
x	300	20	7								
4	1200	80	28								

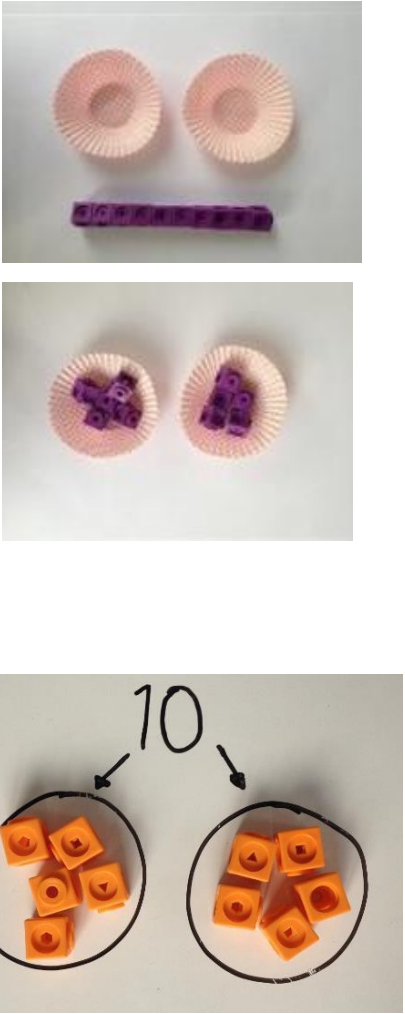
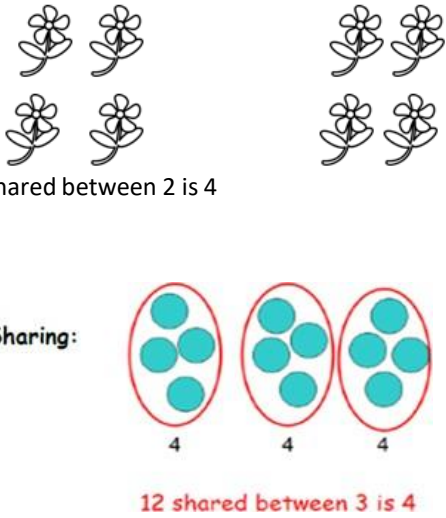
Y5-6

MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract
Column Multiplication for 3 and 4 digits x 1 digit.	Manipulatives may still be used with the corresponding long multiplication modelled alongside.		 
Column multiplication	Manipulatives may still be used with the corresponding long multiplication modelled alongside.		
		Continue to use bar modelling to support problem solving	

Y6 MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract
Multiplying decimals up to 2 decimal places by a single digit.			Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer. $ \begin{array}{r} \overset{1}{3}.\overset{7}{1}9 \\ \times \quad 8 \\ \hline \underline{25.52} \end{array} $

Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing <i>Use Gordon ITPs for modelling</i>	 I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  8 shared between 2 is 4 Sharing: 12 shared between 3 is 4	12 shared between 3 is 4

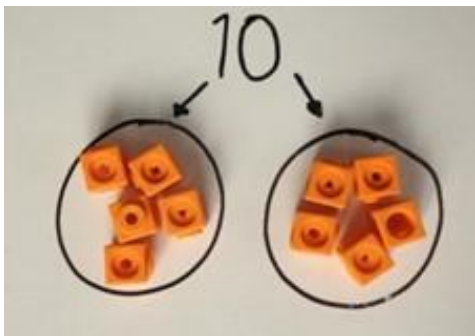
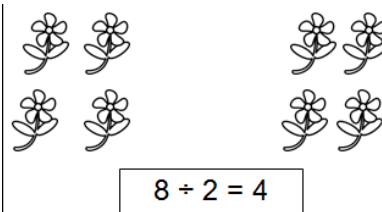
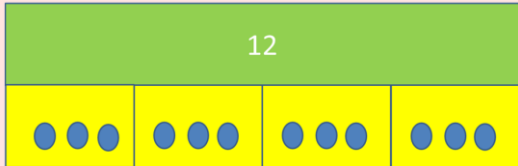
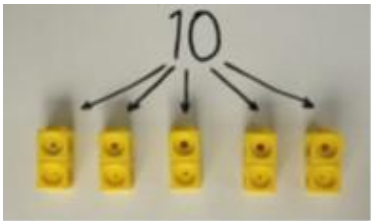
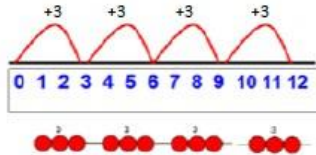
Y1

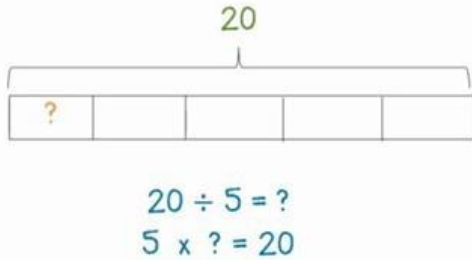
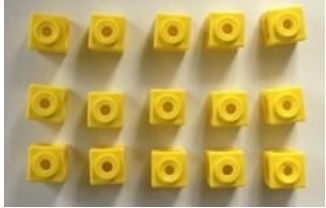
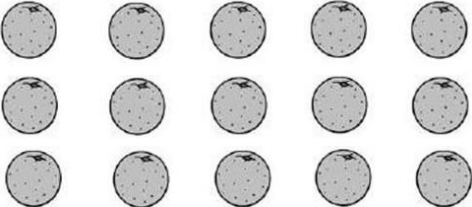
DIVISION ÷

Y2

DIVISION

÷

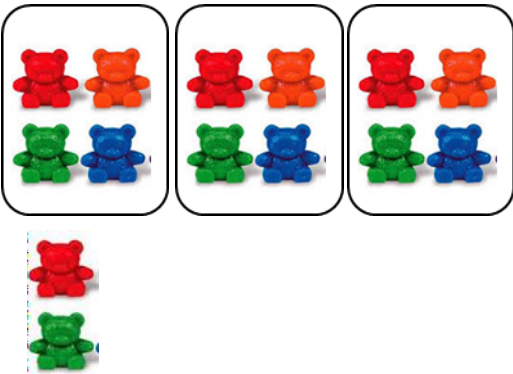
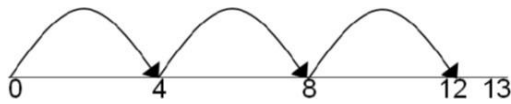
Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing	 I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  $8 \div 2 = 4$ Children use bar modelling to show and support understanding.  $12 \div 4 = 3$	$12 \div 3 = 4$
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  	Use number lines for grouping  $12 \div 3 = 4$ Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.	$28 \div 7 = 4$ Divide 28 into 7 groups. How many are in each group?  $20 \div 5 = ?$ $5 \times ? = 20$

Objective & Strategy	Concrete	Pictorial	Abstract
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding. $96 \div 3 = 32$  3 2	Continue to use bar modelling to aid solving division problems.  $20 \div 5 = ?$ $5 \times ? = 20$	How many groups of 6 in 24? $24 \div 6 = 4$ $\begin{array}{r} 32 \\ 3 \overline{)96} \end{array}$
Division with arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$	Draw an array and use lines to split the array into groups to make multiplication and division sentences 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. $7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$ $28 = 7 \times 4$ $28 = 4 \times 7$ $4 = 28 \div 7$ $7 = 28 \div 4$

Y3

DIVISION



Objective & Strategy	Concrete	Pictorial	Abstract
Division with remainders.	$14 \div 3 =$ Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder. 	Complete written divisions and show the remainder using r. $29 \div 8 = 3 \text{ REMAINDER } 5$ ↑ ↑ ↑ ↑ dividend divisor quotient remainder Children use long division before learning about short division. $ \begin{array}{r} 67 \\ 36 \overline{) 2412} \\ \underline{216} \\ 252 \\ \underline{252} \\ 000 \end{array} $

Y3

DIVISION ÷

Y4-6

DIVISION



Objective & Strategy	Concrete	Pictorial	Abstract
Divide at least 3 digit numbers by 1 digit. Short Division	As per previous years, children can use dienes if necessary.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups. Tens Units 3 2 Encourage them to move towards counting in multiples to divide more efficiently.	Short division when students have mastered long division. Begin with divisions that divide equally with no remainder. $\begin{array}{r} 218 \\ 4 \overline{) 872} \end{array}$ Move onto divisions with a remainder. $\begin{array}{r} 86 \text{ r } 2 \\ 5 \overline{) 432} \end{array}$ Finally move into decimal places to divide the total accurately. $\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \end{array}$ $\begin{array}{r} 0663 \text{ r } 5 \\ 8 \overline{) 5309} \end{array}$ Children to use long division first. Transition to short division after mastery.

Short Division with remainders

Step 1—a remainder in the ones

$$\begin{array}{r} \text{h t o} \\ 041 \text{ R}1 \\ \hline 4 \overline{) 165} \end{array}$$

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

$$\begin{array}{r} \text{th h t o} \\ 0400 \text{ R}7 \\ \hline 8 \overline{) 3207} \end{array}$$

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times ($3,200 \div 8 = 400$)

8 goes into 0 zero times (tens).

8 goes into 7 zero times, and leaves a remainder of 7.

Y4-6

DIVISION ÷